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A METHOD FOR COMPUTING LOWER CONFIDENCE LIMITS ON SYSTEM RELIABILITY USING COMPONENT FAILURE DATA WITH UNEQUAL SAMPLE SIZES

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ABSTRACT:

A method is presented for constructing system reliability using component failure data when the sample sizes for testing on the component parts differ greatly. The procedure can be applied to weapons systems as easily as subsystems. No assumptions about failure distributions are made. The accuracy of the procedure was examined by computer simulations and in this manner the procedure has demonstrated high accuracy for cases of practical interest.

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TABLE OF CONTENTS

		Page
1.	Introduction	1
2.	Approximate Confidence Interval Procedure for Series System Reliability	1
2.1.	Explanation of Procedure	1
2.2.	Accuracy of the Lower Confidence Limit	9
2.3	Simulation Results	11
3.	Examples and Conclusions	19
3.1	Conclusions	19
3.2	Examples	20

1. INTRODUCTION

It has become customary today to interpret some confidence intervals on reliability parameters as exact confidence intervals when, in fact, they are not exact due to implicit and/or explicit reasons. A good deal of effort has been expended on obtaining "exact" reliability confidence intervals in lieu of accepting approximate procedures that may be more easy to implement. In recognition of the fact that so-called "exact" procedures are not exact, it would seem that more effort could be directed toward developing confidence interval estimation procedures that are approximate as such from the outset. This report presents a method for obtaining a lower $100(1-\alpha)\%$ confidence limit for the reliability of a series system using component attributes test data. The number of components tested can vary greatly from one type of component to another, and the procedure remains valid. No assumption is made about the failure distribution of the individual components. The versatility of the procedure allows it to be applied to a subassembly or to an entire system such as the FBWMS.

2. APPROXIMATE CONFIDENCE INTERVAL PROCEDURE FOR SERIES SYSTEM RELIABILITY

(2.1.) Explanation of Procedure

Suppose a system consists of k components in logical series, and suppose that the system reliability, R_s, may be expressed as

$$R_s = \prod_{i=1}^{k} p_i$$

where p_i is the true reliability of the i^{th} component. The components may be either continuously operating or cycle type components.

Suppose we put n_i items of component i on test, $i=1,2,\ldots,k$, under the environmental conditions defined in its mission profile, and let each operate until failure or the mission time is reached, whichever occurs first. Letting f_i equal the number of components of type i that did not complete their mission, we define

(2.1.2) $$\hat{p}_i = 1 - \hat{q}_i$$

where

$$\hat{q}_i = \frac{f_i}{n_i} \quad .$$

Also, we define $\hat{R}_s$ by

$$\hat{R}_s = \prod_{i=1}^{k} \hat{p}_i \quad .$$

The random variable $-\ln \hat{R}_s$ is used to obtain the lower confidence bound on R_s. This is accomplished by fitting the distribution of $-\ln \hat{R}_s$ with a two-parameter gamma distribution by the method of moments. We obtain the confidence bound via the gamma distribution.

To continue the development, define

(2.1.3) $$S \equiv -\ln R_s = \sum_{i=1}^{k} \ln (1-q_i) \quad .$$

Expanding the natural logarithm in an infinite series, we have

$$S = -\sum_{i=1}^{k} \left[(-q_i) - \frac{1}{2}(-q_i)^2 + \frac{1}{3} \ldots \right]$$

(2.1.4)
$$= \sum_{i=1}^{k} \sum_{j=1}^{\infty} \frac{q_i^j}{j} \quad .$$

If each q_i is small, the above series can be approximated by the first two terms of the infinite series, called T_i for ease of expression. That is,

(2.1.5)
$$S \doteq \sum_{i=1}^{k} \left[q_i + \frac{q_i^2}{2} \right] \equiv \sum_{i=1}^{k} T_i$$

where T_i is $q_i + q_i^2/2$. It has been shown that the error due to the above truncation is negligible in cases of practical interest.

An unbiased estimator $\hat{T}_i$ for T_i is

(2.1.6)
$$\hat{T}_i = a_i \hat{q}_i + b_i \frac{\hat{q}_i^2}{2}$$

where

(2.1.7)
$$a_i = \frac{2n_i - 3}{2(n_i - 1)} \quad ,$$

(2.1.8)
$$b_i = \frac{n_i}{n_i - 1} \quad ,$$

and, as in (2.1.2)

$$\hat{q}_i = \frac{f_i}{n_i} \quad .$$

An unbiased estimator is considered important here because the $\hat{T}_i$ are to be added to obtain

$$\hat{S} = \sum_{i=1}^{k} \hat{T}_i \quad . \tag{2.1.9}$$

Therefore, a positive (or negative) bias on all of the $\hat{T}_i$ would be undesirable due to the cumulative effect of their sum. It also follows that $\hat{S}$ will be an unbiased estimator of S since it is a linear function of the $\hat{T}_i$, $i=1,2,\ldots,k$.

An approximate value for the variance of $\hat{S}$ can be shown to be

$$\text{Var}(\hat{S}) = \sum_{i=1}^{k} \text{Var}(T_i) \doteq \sum_{i=1}^{k} \frac{\hat{T}_i}{n_i} \quad . \tag{2.1.10}$$

The distribution of $\hat{S}$ is now fitted with a two-parameter gamma family. This is, it is assumed the probability distribution of $\hat{S}$ is given by the density function $f_{\hat{S}}(x,r,\theta)$ defined as

$$f_{\hat{S}}(x,r,\theta) = \begin{cases} \dfrac{1}{\Gamma(r)\theta^r} x^{r-1} e^{-\frac{x}{\theta}} , & x>0,\ r>0,\ \theta>0 \\ 0 \qquad \text{elsewhere.} \end{cases}$$

The fact that $\dfrac{2\hat{S}}{\theta}$ is a χ^2_{2r} random variable is used to obtain a confidence interval for $-\ln R_s$, and, from this, the confidence interval for R_s can be derived.

Since $\hat{S}$ is assumed to be gamma distributed,

$$E[\hat{S}] = \theta r \tag{2.1.11}$$

and

$$\text{Var}[\hat{S}] = \theta^2 r \tag{2.1.12}$$

But, from (2.1.0)

$$E[\hat{S}] = \sum_{i=1}^{k} T_i$$

and

$$\text{Var}[\hat{S}] \doteq \sum_{i=1}^{k} \frac{T_i}{n_i} \quad .$$

Thus, solving the above equations for r and θ, the following expressions are obtained:

$$r = \left(\sum_{i=1}^{k} T_i \right)^2 \Big/ \sum_{i=1}^{k} \frac{T_i}{n_i} \tag{2.1.13}$$

$$\theta = \left(\sum_{i=1}^{k} \frac{T_i}{n_i} \right) \Big/ \sum_{i=1}^{k} T_i \quad . \tag{2.1.14}$$

Thus, an estimator $\hat{r}$ for r is

$$\hat{r} = \left(\sum_{i=1}^{k} \hat{T}_i \right)^2 \Big/ \sum_{i=1}^{k} \frac{\hat{T}_i}{n_i} \quad . \tag{2.1.15}$$

To obtain the confidence interval, the following procedure is used. Since $\frac{2\hat{S}}{\theta}$ is a χ^2_{2r} random variable and $\theta r = -\ln R_s$, the following probability statements can be made:

$$1-\alpha = P\left[\frac{2\hat{S}}{\theta} \geq \chi_{1-\alpha,\, 2r} \right]$$

$$= P\left[\theta r \leq 2\hat{S} \cdot \frac{r}{\chi^2_{1-\alpha,\, 2r}} \right]$$

$$= P\left[-\ln R_s \leq 2\hat{S} \cdot \frac{r}{\chi^2_{1-\alpha,\, 2r}} \right] \quad .$$

The quantity

$$\frac{2r}{\chi^2_{1-\alpha,\ 2r}} \tag{2.1.17}$$

does not vary considerably if $r \geq 3$. As an example, if $\alpha = .10$, for $r = 6$ the ratio is 1.90, and for $r = 8$ the ratio is 1.72. It is observed that r can be varied by a factor of 1/3 and still have nearly the same value. This effect is even less for larger values of r. This suggests that the probability distribution of

$$\frac{[2\hat{r}]}{\chi^2_{1-\alpha,\ [2\hat{r}]}} \quad , \tag{2.1.18}$$

where $[2\hat{r}]$ denotes the smallest integer greater than or equal to $2\hat{r}$, has a very small variance. Therefore, (2.1.17) could be replaced by (2.1.18) in (2.1.16). We then have the following statement:

$$1-\alpha \doteq P\left[-\ln R_s \leq \frac{\hat{S}[2\hat{r}]}{\chi^2_{1-\alpha,\ [2\hat{r}]}}\right] \tag{2.1.19}$$

or

$$1-\alpha \doteq P\left\{R_s \geq \exp\left[\frac{-\hat{S}[2\hat{r}]}{\chi^2_{1-\alpha,\ [2\hat{r}]}}\right]\right\} \tag{2.1.20}$$

That is, a $100(1-\alpha)\%$ lower confidence limit $\hat{R}_{s,L(\alpha)}$ for R_s is

$$\hat{R}_{s,L(\alpha)} = \exp\left[\frac{-\hat{S}[2\hat{r}]}{\chi^2_{1-\alpha,\ [2\hat{r}]}}\right] \quad . \tag{2.1.21}$$

The reader should observe that the lower confidence limit in (2.1.21) is a random variable. If

(2.1.22) $$P[R \geq \hat{R}_{s,L(\alpha)}] = 1-\alpha$$

for all values of R, then in fact $\hat{R}_{s,L(\alpha)}$ is an exact $100(1-\alpha)\%$ lower confidence limit for R. The degree to which (2.1.22) is true reflects directly upon the approximation of $\hat{R}_{s,L(\alpha)}$ as a lower confidence limit for R. Consequently, we need to examine the accuracy of $\hat{R}_{s,L(\alpha)}$ as lower confidence limit. This can be done by computer simulations. The method for doing this is explained in Section 2.3 of this report.

The accuracy of $\hat{R}_{s,L(\alpha)}$ as defined in (2.1.21) was determined via computer simulation. These accuracy studies revealed that $\hat{R}_{s,L(\alpha)}$ was reasonably accurate when system reliability R_s was below .90. Consequently we set out to improve $\hat{R}_{s,L(\alpha)}$. The basic reasons for the inaccuracies of this lower confidence limit lies in the fact that we have fitted a continuous distribution to $-\ln \hat{R}$. This inherent inaccuracy can be removed in part by making a "continuity correction". This type of correction has a smoothing affect upon the probability distribution of $\hat{R}_{s,L(\alpha)}$. We now proceed to discuss the continuity correction procedure.

Briefly, the continuity correction is made as follows:

1. Determine that component i_0 which has the largest sample size; i.e., $n_{i_0} \geq n_i$, $i=1,\ldots,k$.

2. Define $\hat{T}'_{i_0}$ as

$$\hat{T}'_{i_0} = a_{i_0} \frac{(f_{i_0} + 1)}{n_{i_0}} + \frac{1}{2} b_{i_0} \frac{(f_{i_0} + 1)^2}{n_{i_0}^2} \quad . \tag{2.1.23}$$

That is, we add one more failure to that component with largest sample size to obtain $\hat{T}'_{i_0}$.

3. Define $\hat{T}^*_{i_0}$ by

$$\hat{T}^*_{i_0} = \frac{1}{2} (\hat{T}'_{i_0} + \hat{T}_{i_0}) \quad .$$

4. Substitute $\hat{T}^*_{i_0}$ for $\hat{T}_{i_0}$ in $\hat{S}$ to obtain $\hat{S}^*$. The resulting $\hat{S}^*$ is the "continuity corrected" value of $\hat{S}$.

An equivalent expression for $\hat{S}^*$ is

$$\hat{S}^* = \hat{S} + C \qquad \text{where}$$

$$C = \frac{n_{i_0} + f_{i_0} - 1}{2n_{i_0}(n_{i_0} - 1)} \quad .$$

5. $\hat{r}$ is "corrected" to obtain $\hat{r}^*$ by substituting $\hat{T}^*_{i_0}$ for $\hat{T}_{i_0}$ in the definition of $\hat{r}$.

6. With these definitions of $\hat{S}^*$ and $\hat{r}^*$ we obtain the new $100(1-\alpha)\%$ lower confidence limit $\hat{R}^*_{s, L(\alpha)}$ for R_s given by

$$\hat{R}^*_{s, L(\alpha)} = \exp \{ -\hat{S}^* [2\hat{r}] / \chi^2_{1-\alpha, [2\hat{r}^*]} \} \quad . \tag{2.1.24}$$

2.2 ACCURACY OF THE LOWER CONFIDENCE LIMIT.

The procedure described in Section 2.1 is approximate. The question we should answer now is, "How accurate is the procedure and what are some of its statistical properties?" There are three characteristics of $\hat{R}^*_{s,L(\alpha)}$ which we shall examine to answer this question. They are

1. Mean of $\hat{R}^*_{s,L(\alpha)}$,
2. Variance of $\hat{R}^*_{s,L(\alpha)}$,

and

3. The $(1-\alpha)^{th}$ percentile point of the distribution of $\hat{R}^*_{s,L(\alpha)}$.

The variance is important because we want to have some assurance that the actual values we are likely to come up with when we apply the procedure will not have high probability of being "out of the ball park." The values for 1), 2), and 3) are presented in the tables of simulation results in Section 2.3.

To recognize the importance of the $(1-\alpha)^{th}$ percentile point of $\hat{R}^*_{s,L(\alpha)}$, one only need examine the definition of $\hat{R}^*_{s,L(\alpha)}$ as a lower $100(1-\alpha)\%$ confidence limit for R_s ; i.e.,

$$1-\alpha = P[\hat{R}^*_{s,L(\alpha)} \leq R_s] \quad . \tag{2.2.1}$$

This says that R_s is always the $(1-\alpha)^{th}$ percentile point of the probability distribution of $\hat{R}^*_{s,L(\alpha)}$. Thus if we construct the distribution of $\hat{R}^*_{s,L(\alpha)}$ by computer simulation, we should find that the

$(1-\alpha)^{th}$ percentile point of our constructed distribution is R_s, *if* equation (2.2.1) is correct; i.e., *if* in fact $\hat{R}^*_{s,L(\alpha)}$ is a true $100(1-\alpha)\%$ lower confidence limit for R_s.

We can construct the distribution of $\hat{R}^*_{s,L(\alpha)}$ by generating a large number of random observations on $\hat{R}^*_{s,L(\alpha)}$ for a given set of parameter values $k, n_1, \ldots, n_k, \quad p_1, \ldots, p_k$. That is, the probability distribution of $\hat{R}^*_{s,L(\alpha)}$ is a function of

k: the number of components in the system.

n_i, $i=1,2,\ldots,k$: the number of items of component i tested.

p_i, $i=1,2,\ldots,k$: the reliability of component i. Note these values determine R_s.

Thus we presume to know all of these parameters, and we then examine how well our procedure works; i.e., how accurate it is for this given set of parameters. The accuracy is quantitatively assessed by randomly generating 500 values of $\hat{R}^*_{s,L(\alpha)}$ for the given set of parameter values and computing the mean, the variance, and the $(1-\alpha)^{th}$ percentile point of $\hat{R}^*_{s,L(\alpha)}$. We shall let

(2.2.2) $A_{1-\alpha}$ = $(1-\alpha)^{th}$ percentile point of the distribution of $\hat{R}^*_{s,L(\alpha)}$.

The reader should note that if our assertion in (2.2.1) is correct, then $A_{1-\alpha}$ should equal R_s, regardless of the set of parameter

values we choose and for each value of α we choose. The actual simulation; i.e., generation of the 500 values of $\hat{R}^*_{s,L(\alpha)}$, is easily done on a computer.

Thus the quantity

(2.2.3) $$|A_{1-\alpha} - R_s|$$

is a measure of the accuracy of the procedure.

2.3 SIMULATION RESULTS

The accuracy of the procedure was examined for a variety of sets of parameter values. The different sets of parameters are called cases and are numbered and described in Table 2.3.1. Where the accuracy results for each case appear, the mean and standard deviation of $\hat{R}^*_{s,L(\alpha)}$ are also given in these tables. These values of means and standard deviations are actually the sample means and sample standard deviations of the 500 randomly generated values of $\hat{R}^*_{s,L(\alpha)}$.

The reader should recall that the theoretical values of $A_{1-\alpha}$ should always equal R_s if the procedure is exact for the given set of parameter values. The reader should also keep in mind that $A_{1-\alpha}$ is <u>not</u> the lower confidence limit. The lower confidence limit is $\hat{R}^*_{s,L(\alpha)}$, which is defined in (2.1.24). For example, in Case 1 we see that $R_s = .7233$ and $A_{.95} = .7184$. This represents fairly good

accuracy. That is, for the set of parameter values given in Case 1, $\hat{R}^{*}_{s,L(.05)}$ is very close to a 95% confidence limit for R_s. Likewise, for Case 1, $A_{.90} = .7270$ and $A_{.80} = .729$. These two values differ from R_s by .004 and .006, respectively. This represents quite good accuracy. Thus the statement

$$P[R_s \geq \hat{R}^{*}_{s,L(\alpha)}] = 1-\alpha$$

is very nearly true for $\alpha = .05$, .10, and .20 for the set of parameter values "close" to those given in Case 1.

Case 1 is interesting because of its variety in sample sizes n_i and component reliabilities p_i.

The accurracy of some other life testing procedures is a function of the amount of testing relative to the unreliability or failure rate. In our cases this quantity is expressed by

$$\sum_{i=1}^{k} n_i q_i \quad .$$

This quantity is labeled TT in Table 3.2.1. The results of the cases in Table 2.3.1 imply that this procedure is more accurate as TT increases. More explicit notions about TT are discussed in Section 3.

A reader may desire to use the mean and variance listed in Table 2.3.1 to establish a prediction region within which $\hat{R}^{*}_{s,L(\alpha)}$ is likely to fall. When doing this, one should use the log-normal distribution for $\hat{R}^{*}_{s,L(\alpha)}$.

TABLE 2.3.1

ACCURACY OF $\hat{R}^{*}_{S,L(\alpha)}$ AS A $100(1-\alpha)\%$ LOWER CONFIDENCE LIMIT FOR R_S

Case No.	k	n_i and p_i	TT	R_S	α	$A_{1-\alpha}$	Mean of $\hat{R}^{*}_{S,L(\alpha)}$	Standard Deviation of $\hat{R}^{*}_{S,L(\alpha)}$
1	13	n_i: 150 90 75 100 125 18 28	13.7	.723	.05	.747	.555	.124
		p_i: .995 .985 .979 .988 .982 .980 .967			.10	.735	.599	.115
					.20	.734	.646	.104
		n_i: 125 63 125 59 5 19						
		p_i: 995 .970 .995 .968 .980 .900						
2	15	n_i: 250 40 120 15 130 65 70		.860	.05	.862	.749	.079
		p_i: .990 .990 .990 .990 .990 .990 .990	12.6		.10	.866	.779	.071
					.20	.866	.810	.063
		n_i: 75 100 90 60 60 20 30						
		p_i: .990 .990 .990 .990 .990 .990 .990						
3	15	n_i= 20, i=1, 2,...,15	1.7	.792	.05	.750	.584	.089
		p_i=.995, i=1, 2,...,14; p_{15}= .85			.10	.759	.645	.993
					.20	.768	.705	.092
4	15	n_i= 40, i=1, 2,...,15		.792	.05	.768	.660	.062
		p_i=.995, i=1, 2,...,14; p_{15}= .85			.10	.782	.697	.063
					.20	.793	.736	.062

TABLE 2.3.1 Continued

Case No.	k	n_i and p_i	TT	R_S	α	$A_{1-\alpha}$	Mean of $\hat{R}^*_{S,L(\alpha)}$	Standard Deviation of $\hat{R}^*_{S,L(\alpha)}$
5	15	n_i= 50, i=1,2,...,15	4.3	.792	.05	.770	.672	.059
		p_i= .995, i=1,2,...,14; p_{15}= .85			.10	.750	.704	.060
					.20	.790	.738	.059
6	15	n_i= 100, i=1,2,...,15						
		p_i= .995, i=1,2,...,14; p_{15}= .85	8.5	.792	.05	.781	.717	.041
					.10	.791	.737	.041
					.20	.792	.760	.040
7	15	n_i= 150, i=1,2,...,5; n_i= 20, i=6,...,15	5.0	.792	.05	.733	.586	.088
		p_i= .995, i=1,2,...,14; p_{15}= .850			.10	.746	.646	.092
					.20	.787	.706	.092
8	15	n_i= 20, i=1,2,...,14; n_{15}= 150	3.6	.792	.05	.811	.677	.081
		p_i= .995, i=1,2,...,14; p_{15}= .850			.10	.813	.767	.074
					.20	.804	.739	.067
9	5	n_i= 50, i=1,2,...,5			.05	.996	.856	.054
		p_i= .990, i=1,2,...,5	2.5	.951	.10	.927	.887	.048
					.20	.952	.914	.042

TABLE 2.3.1 Continued

Case No.	k	n_i and p_i	TT	R_S	α	$A_{1-\alpha}$	Mean of $\hat{R}^*_{S,L(\alpha)}$	Standard Deviation of $\hat{R}^*_{S,L(\alpha)}$
10	10	n_i= 50, i=1,2,...,10		.904	.05	.893	.802	.050
		p_i=.990, i=1,2,...,10	5.0		.10	.900	.834	.050
					.20	.901	.864	.047
11	15	n_i= 50, i=1,2,...,15	7.5	.860	.05	.816	.745	.050
		p_i=.990, i=1,2,...,15			.10	.847	.777	.050
					.20	.857	.810	.050
12	5	n_i= 100, i=1,2,...,5	2.5	.774	.05	.763	.692	.042
		p_i= .95, i=1,2,...,5			.10	.767	.713	.042
					.20	.770	.736	.041
13	25	n_i= 25, i=1,2,...,25	3.1	.882	.05	.985	.717	.101
		p_i= .995, i=1,2,...,25			.10	.858	.759	.094
					.20	.906	.816	.085
14	25	n_i= 50, i=1,2,...,25	6.2	.882	.05	.863	.770	.053
		p_i= .995, i=1,2,...,25			.10	.871	.802	.054
					.20	.878	.833	.052

TABLE 2.3.1 Continued

Case No.	k	n_i and p_i	TT	R_S	α	$A_{1-\alpha}$	Mean of $\hat{R}^*_{S,L(\alpha)}$	Standard Deviation of $\hat{R}^*_{S,L(\alpha)}$
15	25	n_i= 100, i=1,2,...,25	12.4	.882	.05	.869	.812	.035
		p_i= .995, i=1,2,...,25			.10	.876	.831	.035
					.20	.878	.833	.052
16	50	n_i= 25, i= 1,2,...50	6.3	.778	.05	.702	.594	.076
		p_i= .995, i = 1,2,...,50			.10	.719	.644	.079
					.20	.771	.695	.079
17	50	n_i= 50, i=1,2,...,50	12.6	.778	.05	.755	.660	.061
		p_i= .995, i=1,2,...,50			.10	.767	.691	.062
					.20	.777	.725	.061
18	50	n_i= 100, i=1,2,...,50	25.2	.778	.05	.757	.697	.040
		p_i= .995, i=1,2,...,50			.10	.769	.717	.040
					.20	.773	.739	.040
19	100	n_i= 25, i=1,2,...,100	12.5	.606	.05	.569	.438	.077
		p_i= .995, i=1,2,...,100			.10	.588	.480	.081
					.20	.608	.527	.084
20	100	n_i= 50, i=1,2,...,100		.606	.05	.600	.492	.056
		p_i= .995, i=1,2,...,100	25		.10	.604	.520	.060
					.20	.598	.553	.062

TABLE 2.3.1 Continued

Case No.	k	n_i and p_i	TT	R_S	α	$A_{1-\alpha}$	Mean of $\hat{R}^*_{S,L(\alpha)}$	Standard Deviation of $\hat{R}^*_{S,L(\alpha)}$
21	100	n_i= 100, i=1,2,...,100	50	.606	.05	.588	.526	.040
		p_i= .995, i=1,2,...,100			.10	.594	.545	.041
					.20	.604	.566	.041
22	25	n_i= 25, i=1,2,...,25	.6	.975	.05	.985	.888	.104
		p_i= .999, i=1,2,...,25			.10	.992	.921	.078
					.20	.996	.946	.056
23	25	n_i= 50, i=1,2,...,25	1.3	.975	.05	.996	.904	.058
		p_i= .999, i=1,2,...,25			.10	.998	.929	.045
					.20	.999	.950	.034
24	25	n_i= 100, i=1,2,...,25	2.5	.975	.05	.999	.926	.030
		p_i= .999, i=1,2,...,25			.10	.963	.943	.025
					.20	.976	.957	.023
25	50	n_i= 25, i=1,2,...,50	1.3	.951	.05	.985	.822	.106
		p_i= .999, i=1,2,...,50			.10	.992	.868	.085
					.20	.996	.905	.066
26	50	n_i= 50, I=1,2,...,50	2.5	.951	.05	.996	.857	.055
		p_i= .999, i=1,2,...,50			.10	.927	.887	.049
					.20	.952	.913	.043

TABLE 2.3.1 Continued

Case No.	k	n_i and p_i	TT	R_S	α	$A_{1-\alpha}$	Mean of $\hat{R}^*_{S,L(\alpha)}$	Standard Deviation of $\hat{R}^*_{S,L(\alpha)}$
27	50	n_i= 100, i=1,2,...,50		.951	.05	.929	.892	.028
		p_i= .999, i=1,2,...,50	5.0		.10	.947	.909	.027
					.20	.949	.926	.026
28	100	n_i= 25, i=1,2,...,100	2.5	.905	.05	.985	.736	.093
		p_i= .999, i=1,2,...,100			.10	.858	.790	.085
					.20	.906	.837	.076
29	100	n_i= 50, i=1,2,...,100	5.0	.905	.05	.863	.793	.048
		p_i= .999, i=1,2,...,100			.10	.896	.826	.048
					.20	.901	.856	.046
30	100	n_i= 100, i=1,2,...,100	10.0	.905	.05	.891	.837	.033
		p_i= .999, i=1,2,...,100			.10	.898	.855	.033
					.20	.904	.874	.032

3. EXAMPLES AND CONCLUSIONS

3.1 Conclusions

Examination of Table 2.3.1 reveals that the accuracy of this procedure appears to be a function of the quantity

$$TT \equiv \sum_{i=1}^{k} n_i q_i \quad . \tag{3.1.1}$$

This quantity is the amount of testing relative to the component unreliabilities. Table 3.1.1 gives minimum and reasonable values of TT for which $\hat{R}^*_{s,L(\alpha)}$ as defined in (2.1.24) is a reasonably accurate lower confidence limit for system reliability. The values of TT needed for reasonable accuracy varies with the desired confidence level as can be seen from Table 3.1.1. The numbers in Table 3.1.1 were obtained by visual inspection of Table 2.3.1. The reader may prefer different values for Table 3.1.1 than those constructed by the authors.

TABLE 3.1.1

Values of TT Needed for Reasonable Accuracy of $\hat{R}^*_{s,L(\alpha)}$ as a Lower $100(1-\alpha)\%$ Confidence Limit for System Reliability, R_s .

$100(1-\alpha)\%$	Minimum TT	Reasonable TT
80	3	4
90	8	10
95	15	20

In addition to the restraints on TT given in Table 3.1.1, it is also necessary to put minimum restraints on the sample sizes n_i for each component in the system. As an extreme example, suppose

a series system consists of 10 components and 10,000 items of component one are tested and none of the remaining components are tested. This testing program may give a "large" value to TT but is certainly an unsatisfactory testing program. What is needed are additional restrictions on the minimum sample sizes of each type of component in the system. We recommend that a minimum sample size of 10 be allowed for each component in the series system when an 80% confidence limit is desired.

The reader can take a more cautious attitude by simulating any particular series system he is confronted with to establish the accuracy of this procedure for his particular situation. That is, if the reader has series system where the number k of components in the system is known, the sample sizes n_i on each of the components are known, and the reliability goals on each of the components are known; then he can simulate testing for these parameter values to determine the accuracy of the proposed lower confidence limit in (2.1.24) for this particular series system with these given parameter values. This simulation would be exactly the same as that done to obtain the accuracy results displayed in Table 2.3.1.

3.2 Examples

The lower confidence limit $\hat{R}^*_{s,L(\alpha)}$ given in equation (2.1.24) can be used to obtain a lower confidence limit on 1) the entire system reliability such as the FBMWS, or 2) on subsystem reliability, or

3) on assembly reliability, or 4) on components at lower levels of assembly.

The following two examples demonstrate how the procedure is used. One example has a small number of components in the system such as would be the case if the system were the FBMWS and the k components were the subsystems. The second example has a larger number of components.

Example 1.

A system consists of 8 components in logical series. The following data depicts the number of mission trials and consequent failures that have been accumulated against respective components.

TABLE 3.2.1

	Component							
	1	2	3	4	5	6	7	8
n_i: Number of mission trials	101	300	401	200	50	100	101	10
f_i: Number of failures	1	0	3	0	0	0	1	0

The reliability goal for each of the eight components is .995. A lower 80% confidence limit on system reliability is desired.

Since each unreliability goal is .005, if each component meets its reliability goal, the amount of testing TT will be

$$TT = \sum_{i=1}^{8} n_i (.005) = 6.3 \quad .$$

Thus the amount of testing to date is sufficient for the procedure to be reasonably accurate.

With the data from Table 3.2.1 we obtain Table 3.2.2.

TABLE 3.2.2

Component No.	n_i	f_i	a_i	b_i	$\hat{q}_i^*$	$\hat{T}_i^*$
1	101	1	.9950	1.01	.0099	.0099
2	---	-	-----	----	0	0
3	401	3	.9981	1.025	.0100	.0100
4	---	-	-----	-----	0	0
5	---	-	-----	-----	0	0
6	---	-	-----	-----	0	0
7	101	1	.9950	1.01	.0099	.0099
8	---	-	-----	-----	0	0

From Table 3.2.2 we obtain

$$\hat{S}^* = .0298$$

$$\hat{r}^* = 2.74$$

$$[2\hat{r}^*] = 6$$

Also $\chi^2_{.80,6} = 3.070$. Thus

$$\hat{S}^* [2\hat{r}^*]/\chi^2_{.80,[2\hat{r}^*]} = (.0298) \cdot (1.954) = .058 \ .$$

Thus

$$\hat{R}_{s,L(.20)} = e^{-.058} = .942 \ .$$

That is, from the test data in Table 3.2.1 and our formula for $\hat{R}^*_{s,L(.20)}$ we would feel 80% confident that the reliability of the system is better than .942 .

Example 2.

A system consists of 30 items in logical series. From the test data in Table 3.2.3 we wish to construct an 80% lower confidence limit on the system reliability.

TABLE 3.2.3

	Component 1	2	3	All Other Components
n_i	101	401	101	$n_i < 401$
f_i	1	3	1	0

The data in this table will give us the same value of $\hat{R}^*_{s,L(.20)}$ as that in Example 1. That is $\hat{R}^*_{s,L(.20)} = .942$.

The reader may find it strange that this procedure "apparently" does not use the sample sizes of the items that do not fail. We must remember, however, that the value of TT should be at least 3 for the procedure to be accurate. Thus suppose the reliability goal of all components was .997. Suppose also that we tested only 5 items of each of the 27 components which did not fail. Then we would have

$$TT = \sum_{i=1}^{30} n_i q_i = .003[101 + 401 + 101 + 27(5)]$$

$$= 2.2 \quad .$$

This value of TT is too low to justify using this procedure. The authors would recommend the procedure not be used unless we had tested 20 items for each of the 27 different types of components which had not failed. This would make TT = 3.4 .

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13. ABSTRACT

A method is presented for constructing system reliability using component failure data when the sample sizes for testing on the component parts differ greatly. The procedure can be applied to weapons systems as easily as subsystems. No assumptions about failure distributions are made. The accuracy of the procedure was examined by computer simulations and in this manner the procedure has demonstrated high accuracy for cases of practical interest.

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